The impacts of observed small turbulent Lewis number in stable stratification: changes in the thermal production?

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1 Motivations

It is shown in Marquet *et al.* (2017a,b) that the assumption of equality of exchange coefficients K_h for heat, K_w for water and K_s for entropy is not supported by observations.

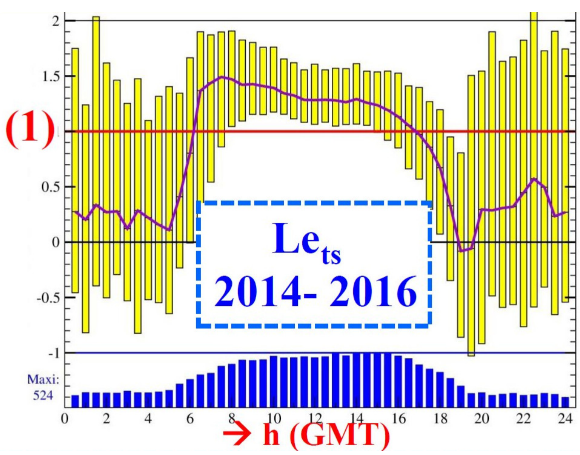

Figure 1: *The boxplots (yellow interquartile range) for the moist-entropy turbulent Lewis number $Le_{ts} = K_s/K_w$ in terms of the GMT hours for the the Météopole-Flux mast and for a 2 years average (CNRM at Toulouse, France). The number of observations for each class of hours are in blue vertical bars. The level $Le_{ts} = 1$ is in red and median values are in purple piecewise curve.*

As an example, Figure 1 shows that yearly average values of the moist-entropy Lewis turbulent number $\text{Le}_{ts} = K_s/K_w$ are significantly larger than unity in daytime, and are lower than 0.5 at night. Moreover, values of Le_{ts} may reach the zero level for stable stratifications (here between 19 and 20 GMT).

The consequences of $\text{Le}_{ts} = K_s/K_w \approx 0$ are derived in this note in terms of the thermal production $\beta\ \overline{w'\theta'_v}$, which is one of the terms forming the turbulent kinetic energy equation, where $\beta = g/\theta_0$ and where the virtual potential temperature is $\theta_v = \theta\ (1 + \delta\, q_v - q_l - q_i)$, with $\delta \approx 0.6$ and $\theta = T\ (p_0/p)^{R_d/c_{pd}}$.

To make simple, and in order to simulate moist and cloud-free conditions like the IHOP case (Couvreux *et al.*, 2005), condensed water will be neglected in this preliminary study, leading to $q_l + q_i = 0$ and to

$$\boxed{\theta_v \ = \ \theta\ (1 \ + \ \delta\, q_v)} \, . \tag{1}$$

The Betts (1973) moist variables are then equal to $\theta_l = \theta$ and $q_t = q_v$ and the first-order approximation of the moist-entropy potential temperature defined in Marquet (2011, 2015, 2016) can be written as

$$\theta_s \ \approx \ \boxed{(\theta_s)_1 \ = \ \theta\ \exp(\Lambda\, q_v)} \, , \tag{2}$$

where $\Lambda \approx 6$ and $(\theta_s)_1 \approx \theta\ (1 + \Lambda\, q_v)$

2 Computation of $\overline{w'\theta'_v}$

The differentials of Eqs. (1) and (2) are

$$d\theta_v \ = (1 + \delta\, q_v)\, d\theta \ + \ \delta\, \theta\, dq_v \, , \tag{3}$$

$$d(\theta_s)_1 \ = \exp(\Lambda\, q_v)\, d\theta \ + \ \Lambda\, (\theta_s)_1\, dq_v \, . \tag{4}$$

From Eq. (4) the differential of θ can be computed as $d\theta = \exp(-\,\Lambda\, q_v)\ d(\theta_s)_1 \ - \ \Lambda\ \theta\ dq_v$. This expression can then be inserted into Eq. (3) to give

$$d\theta_v \ = \ (1 + \delta\, q_v)\ \exp(-\,\Lambda\, q_v)\, d(\theta_s)_1 \\ - \ [\ (\Lambda \ - \ \delta) \ + \ \Lambda\, \delta\, q_v\ ]\ \theta\ dq_v \, . \tag{5}$$

By applying Reynolds hypotheses, the vertical flux $\overline{w'\theta'_v}$ can be computed from Eq. (5) in terms of the vertical fluxes of $(\theta_s)_1$ and q_v, leading to

$$\overline{w'\theta'_v} \ = \ (1 + \delta\, \overline{q_v})\ \exp(-\,\Lambda\, \overline{q_v})\ \overline{w'(\theta'_s)_1} \\ - \ [\ (\Lambda \ - \ \delta) \ + \ \Lambda\, \delta\, \overline{q_v}\ ]\ \overline{\theta}\ \ \overline{w' q'_v} \, , \tag{6}$$

$$\text{and} \quad \overline{w'\theta'_v} \ \approx \ \overline{w'(\theta'_s)_1} \ - \ 5.4\ \overline{\theta}\ \ \overline{w' q'_v} \, . \tag{7}$$

The approximate flux (7) is obtained with the assumptions $1 \ + \ 0.6\ \overline{q_v} \ \approx \ 1$, $\exp(-\ 6\ \overline{q_v}) \ \approx \ 1$ and $6 \ \times \ 0.6 \ \ \overline{q_v} \ \ll 6 - 0.6 \ \approx \ 5.4$. The same approximation can be used to derive a similar expression for the vertical derivatives, leading to

$$\frac{\partial \overline{\theta_v}}{\partial z} \ \approx \ \frac{\partial \overline{(\theta_s)_1}}{\partial z} \ - \ 5.4\ \overline{\theta}\ \ \frac{\partial \overline{q_v}}{\partial z} \, . \tag{8}$$

3 $\overline{w'\theta'_v}$ expressed in terms of Le_{ts}

According to Richardson (1919), the turbulence is applied to the total water content ($q_t = q_v$) and to the moist entropy variable $(\theta_s)_1$. Accordingly, it is assumed that the vertical fluxes of $(\theta_s)_1$ and q_v can be expressed in terms of the (positive) exchange coefficients K_s and K_w, leading to

$$\overline{w'(\theta'_s)_1} \approx -K_s \frac{\partial \overline{(\theta_s)_1}}{\partial z} = -K_w \, \mathrm{Le}_{ts} \frac{\partial \overline{(\theta_s)_1}}{\partial z} , \quad (9)$$

$$\overline{w' q'_v} \approx -K_w \frac{\partial \overline{q_v}}{\partial z} , \quad (10)$$

where $\mathrm{Le}_{ts} = K_s / K_w$ is the turbulent Lewis number.

Eqs. (9) and (10) can then be inserted into (7), leading to the two alternative formulations:

$$\overline{w'\theta'_v} \approx -K_w \left[\mathrm{Le}_{ts} \frac{\partial \overline{(\theta_s)_1}}{\partial z} - 5.4 \; \overline{\theta} \; \frac{\partial \overline{q_v}}{\partial z} \right] \quad (11)$$

or, from (8):

$$\overline{w'\theta'_v} \approx -K_w \left[\frac{\partial \overline{\theta_v}}{\partial z} + (\mathrm{Le}_{ts} - 1) \frac{\partial \overline{(\theta_s)_1}}{\partial z} \right] . \quad (12)$$

The second formulation (12) shows that the assumptions $\mathrm{Le}_{ts} = 1$ and $K_s = K_h = K_w$ (made in all present RCMs, NWP models and GCMs parameterizations of turbulence) correspond to a cancellation of the second term into brackets, and then to

$$\boxed{ \left(\overline{w'\theta'_v} \right)_{\mathrm{Le}_{ts}=1} \approx -K_w \frac{\partial \overline{\theta_v}}{\partial z} = -K_h \frac{\partial \overline{\theta_v}}{\partial z} . } \quad (13)$$

For very stable conditions and for the present assumption $\mathrm{Le}_{ts} = 1$, then $\partial \overline{\theta_v} / \partial z \gg 0$ and the thermal production $\beta \; \overline{w'\theta'_v}$ is negative due to $\beta = g/\theta_0 > 0$ and to $-K_w < 0$. These negative values for $\overline{w'\theta'_v}$ lead to a rapid extinction of turbulence via the turbulent kinetic energy equation $\partial e / \partial t = \beta \; \overline{w'\theta'_v} + \ldots$.

Differently, values $\mathrm{Le}_{ts} = K_s / K_w \approx 0$ observed in Figure 1 at night and for stable conditions can be inserted into the first formulation (11), leading to a cancellation of the first term into brackets and to

$$\boxed{ \left(\overline{w'\theta'_v} \right)_{\mathrm{Le}_{ts}=0} \approx +K_w \; (\, 5.4 \; \overline{\theta} \,) \; \frac{\partial \overline{q_v}}{\partial z} . } \quad (14)$$

This is a drastic and very important change in the nature of turbulence in stable conditions: if $\mathrm{Le}_{ts} \approx 0$, the thermal production no longer depends on $\partial \overline{\theta_v} / \partial z$ and, rather, $\beta \; \overline{w'\theta'_v}$ only depends on the sign of the vertical gradient of water vapour content $\partial \overline{q_v} / \partial z$!

The physical consequence is that, for $\mathrm{Le}_{ts} \approx 0$, the turbulence may be maintained despite positive values of $\partial \overline{\theta_v} / \partial z$ (which have no impact) and precisely in those regions where $\partial \overline{q_v} / \partial z$ is positive.

Intermediate values of Le_{ts} between 0 and 1 (or above 1) would imply a mixed influence of the two vertical gradients of $\overline{q_v}$ and $\overline{\theta_v}$.

4 Conclusion

Observations of small or null values of the turbulent Lewis number $\mathrm{Le}_{ts} = K_s / K_w$ at night and in stable and moist conditions may have a large impact on the thermal production and the vertical flux $\overline{w'\theta'_v}$.

It is shown that $\mathrm{Le}_{ts} \approx 0$ means that $\overline{w'\theta'_v}$ becomes proportional to (and is of the same sign as) the vertical gradient of water vapour content $\partial \overline{q_v} / \partial z$, with a factor ($5.4 \; \overline{\theta}$) which is of the order of $5.4 \times 300 \approx 1600$. This means that a moderate vertical change of $\Delta \overline{q_v} = +1$ g/kg in (14) would have a large impact corresponding to positive and significant values of thermal production, an impact similar to the one created by an unstable value $\Delta \overline{\theta_v} = -1.6$ K in (13) for $\mathrm{Le}_{ts} = 1$.

The behaviour of present turbulent kinetic equations might be improved by taking into account Eq. (14) in stable and moist conditions. It is thus needed to revisit the theoretical formulations of K_s and K_w in existing RCMs, NWP models and GCMs parameterizations of turbulence, with the need to represent Lewis numbers different from 1 and depending on the local stability.

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