On consequences of measurements of turbulent Lewis number from observations.

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1 Motivations

Almost all parameterizations of turbulence in NWP models and GCM make the assumption of equality of exchange coefficients K_h for heat and K_w water. These two exchange coefficients are applied to the two moist-air Betts (1973) "conservative" variables

$$\theta_l = \theta \exp\left(-\frac{L_v q_l + L_s q_i}{c_{pd} T}\right), \quad (1)$$

$$q_t = q_v + q_l + q_i \quad (2)$$

(where $\theta = T (p_0/p)^{R_d/c_{pd}}$ is the dry air potential temperature) to compute the vertical turbulent fluxes written as: $\overline{w'\theta_l'} = -K_h \partial\overline{\theta_l}/\partial z$ and $\overline{w' q_t'} = -K_w \partial\overline{q_t}/\partial z$.

However, large uncertainties exists in old papers published in the 1950s, 1960s and 1970s, where the turbulent Lewis number $\mathrm{Le}_t = K_h/K_w$ have been evaluated from observations. Some papers are favourable to the hypothesis $K_h = K_w$ and $\mathrm{Le}_t = 1$, while others have observed higher values, up to $\mathrm{Le}_t > 4$.

Moreover, the use of the Betts variable θ_l is based on an approximate moist-air entropy equation and this formulation has been improved in Hauf and Höller (1987) and Marquet (2011, 2015), where the new potential temperature θ_s is defined as synonymous of the moist-air entropy.

The aim of this note is: 1) to trust the recommendations of Richardson (1919), who suggested to use the moist-air entropy as a variable on which the turbulence is acting; 2) then to replace θ_l by the third-law entropy value θ_s, which must correspond to a new exchange coefficients K_s; 3) compute K_s and $\mathrm{Le}_{ts} = K_s/K_w$ from observations (Météopole-Flux and Cabauw masts) and from LES and SCM outputs for the IHOP case (Couvreux *et al.*, 2005).

2 The moist-air entropy flux

The specific (per unit mass of *moist-air*) entropy is defined in Marquet (2011, 2015) by $s = s_{ref} + c_{pd} \ln(\theta_s)$, where s_{ref} and c_{pd} are two constants. If liquid water or ice do not exist, $\theta_l = \theta$ and the first-order approximation of the moist-air entropy potential temperature is $\theta_s \approx \theta \exp(\Lambda q_v)$, where $\Lambda \approx 5.87$ is a constant which depends on the third-law reference values of entropy of dry air and water vapour. The second-order approximation derived in Marquet (2016) writes

$$\theta_s \approx \theta \exp(\Lambda q_v) \exp[-\gamma \ln(r_v/r_*) q_v], \quad (3)$$

where $\gamma \approx 0.46$ and $r_* \approx 12.4$ g/kg are two constants.

With Reynolds hypotheses, the flux of moist-air entropy potential temperature can be written as

$$\overline{w'\theta_s'} = -K_s \frac{\partial\overline{\theta_s}}{\partial z}, \quad (4)$$

$$\overline{w'\theta_s'} \approx \exp(\Lambda \overline{q_v}) \, \overline{w'\theta'}$$
$$+ \overline{\theta_s}\left[\Lambda - \gamma \ln\left(\frac{\overline{r_v}}{r_*}\right) - \frac{\gamma}{1-\overline{q_v}}\right] \overline{w'q_v'}. \quad (5)$$

This flux is a weighted sum of the fluxes for θ and q_v. And if the turbulence is to be represented by the flux of θ_s and q_v, the corresponding flux of θ is given by

$$\overline{w'\theta'} \approx -K_w \, \mathrm{Le}_{ts} \, \frac{\partial\overline{\theta}}{\partial z} \quad (6)$$
$$- K_w \, (\mathrm{Le}_{ts} - 1)\left[\Lambda - \gamma \ln\left(\frac{\overline{r_v}}{r_*}\right) - \frac{\gamma}{1-\overline{q_v}}\right] \overline{\theta} \, \frac{\partial\overline{q_v}}{\partial z},$$

where the moist-entropy Lewis turbulent number is $\mathrm{Le}_{ts} = K_s/K_w$.

If $\mathrm{Le}_{ts} = 1$, the second line of (6) cancels out and $K_h = K_s = K_w$ allows to write the flux of θ as $\overline{w'\theta'} = -K_h \partial\overline{\theta}/\partial z$, in terms of the exchange coefficient K_h.

Differently, if $\mathrm{Le}_{ts} \neq 1$, the second line of (6) exists and the flux of θ is not proportional to the sole vertical gradient of θ: it also depends on the vertical gradient of q_v. This prevents defining properly an "exchange coefficient K_h for θ", and the turbulence must clearly be applied to θ_s and q_v, and not to $\theta_l = \theta$ and q_v.

It is thus important to try to determine, from observations and/or from numerical results, whether $\mathrm{Le}_{ts} = 1$ or if Le_{ts} is significantly different from unity?

3 Results

Figures 1 show that average yearly Lewis turbulent numbers computed with the the eddy-correlation method are significantly larger than unity in daytime, and are lower than 1.0 at night. The significant level is more often reached for monthly averages Figures (not shown) and this diurnal cycle is also observed almost each days, with a maximum present just after sunrise.
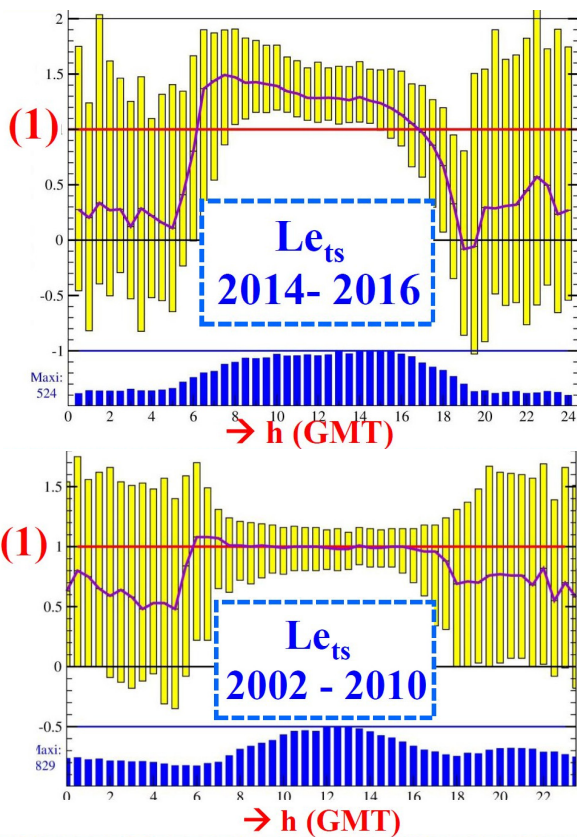
Figure 1: *The boxplots for the Lewis number $Le_{ts} = K_s/K_w$ in terms of the GMT hours. Top: the Météopole-Flux mast for 2 years (CNRM at Toulouse, France). Bottom: thanks to F. Bosvelt, the Cabauw mast for 9 years (KNMI at Utrecht, Holland). The number of observations for each class of GMT hours are in blue vertical bars. The level $Le_{ts} = 1$ is in red and median values are in purple piecewise curve.*

This maximum of Le_{ts} is often larger than 2 in June-August and if often smaller than 0.8 in winter.

The observed diurnal cycle for Le_{ts} may explain the previous disagreements in the articles of the 1970s: values close to 1.0 may be observed in the late afternoon and values larger than unity in the early morning?

Figure 2 shows that the LES outputs for the IHOP case lead to robust computations of the new moist-air entropy exchange coefficient $K_s = -(\overline{w'\,\theta'_s})/(\partial \overline{\theta_s}/\partial z)$ and of K_w by means of the eddy-correlation method, whereas values of K_h determined from the flux and the vertical gradient of θ is subject to infinite values (due to zero vertical gradient from about 150 to 300 m, depending on the hour) and to a counter-gradient region (due to the same signs of flux and vertical gradient above the level of infinite value).

The turbulent Lewis number is close to 1.0 close to the surface on Figure 2, and then increases with altitude, reaching values above 1.5 above the 150 m height where the mass-flux starts to be active.

4   Conclusion

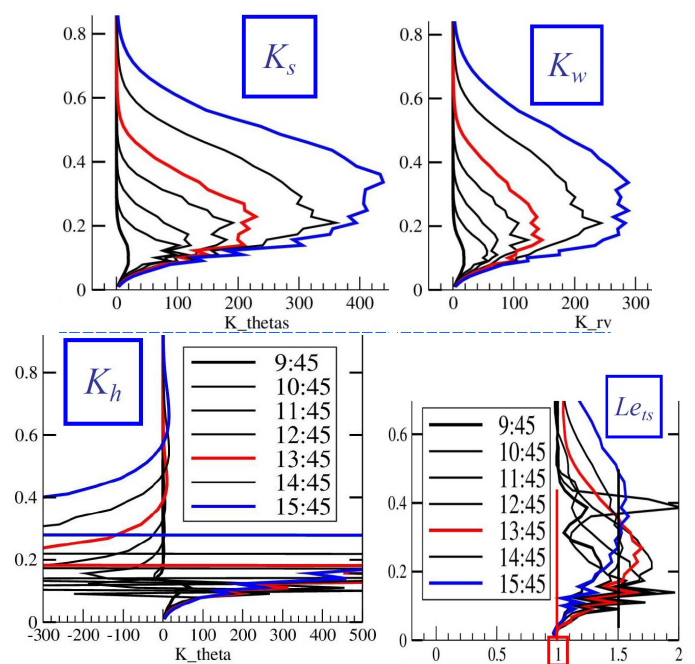Values of the turbulent Lewis number significantly smaller or larger than 1.0 are observed for the

Figure 2: *The vertical profiles of K_s, K_w, K_h and $Le_{ts} = K_s/K_w$ for hourly averages of a LES run for the moist (but clear-air) case IHOP. Heights are in ordinates from 0 to 6 km or 8 km.*

Météopole-Flux and the Cabauw masts, and simulated by a LES of the IHOP case. Consequently, it is necessary to revisit the equations of turbulence and to determined the values of several new constant coefficients (for the pressure and the second-order moment terms). These observations and simulations outputs will be used to compute these coefficients.

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